

Active Learning Model and Students' Engagement in Teaching Mathematics:**Development and Implementation**

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Abstract

This study aimed to develop and implement an active learning model for teaching mathematics at the secondary level. The active learning model (4WsHs) for teaching mathematics was developed after studying related literature, existing models for active learning, and active learning strategies for teaching mathematics. The model was validated by the committee of experts using three phases of the Delphi technique. To test the effectiveness of the newly developed active learning model, an experiment was conducted by following a pretest-posttest equivalent groups design. The students of 9th grade in two Government High Schools, Mansehra were taken as a sample of the study. The students were divided into two equivalent groups as control and experimental groups based on pre-test scores by using matched random sampling technique. The experimental group received treatment by teaching through the 4WsHs model while the control group was taught through the traditional chalk and talk method. The findings elicited that the new active learning model was effective in enhancing the academic achievement of students in mathematics. The study suggested that teachers may receive training to implement active learning techniques in the Mathematics classroom. Moreover, skilled teachers may improve students' learning outcomes in Mathematics by utilizing the vast arsenal of active learning strategies as suggested in the model.

Keywords: Active Learning, Academic Achievement, Secondary School Students, Teaching Mathematics

Introduction

Teacher effectiveness has become one of the most important school-related factors in students' academic achievement. Teachers practice two modes in the teaching-learning process around the globe; teacher-centered approach and learner-centered approach. During a teacher-centered approach, teaching revolves around the teacher without the involvement of students in the lesson. This approach is effective for short-term recall as it supports rote memorization of disconnected rules (Swan, 2005; Darling-Hammond, Flook, Cook-Harvey, Barron, & Osher, 2019). It facilitates passive learning and there is no space for previous knowledge and feedback. In comparison with the traditional teacher-centered teaching approach, the learner-centered approach is much effective (Abdullah & Yang, 2019). A student-centered approach brings a positive impact on content, activities, materials, and the speed of students' learning. There is a positive collaboration among students and teachers that connects new information to previous knowledge (Corkin, Horn, & Pattison, 2017). Learner-centered pedagogy is a dynamic process that engages students and enables them to build the conceptual framework around the topic (Chika, 2012). Moreover, learner-centered pedagogies involve students in constructing knowledge through participation in classroom activities and reflection (Ahn, & Class, 2011). The learner-centered approach has been built on two concepts; the first is about learning as a dynamic process and the second is learners' learning style (Zabeli, Anderson, & Saqipi, 2018). The movement for learner-centered approach presents the active, dynamic, and cognitive construct of knowledge in its nature. Inactive learning whole learning mechanism revolves around the learners. Every learner has its unique style of learning and presenting and that is the essence of learning-centered pedagogies. According to Stoblein (2009), active learning incorporates learning within a student's previous knowledge by providing different activities to assist learning.

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Active learning philosophical foundations are established on cognitive and behavioral aspects of learning. The cognitive domain of active learning is based on the work of the French psychologist Jean Piaget (1896-1980). It indicates the role of teaching strategies that can engage the learner in different levels of thinking. While the behavioral dimension traces back to the American philosopher John Dewey (1859-1952), who concentrates on learners' engagement during teaching. In theory, active learning relates to constructivism. It is based on the principle that students can construct their knowledge if they are actively engaged in the learning process or adequate interaction and exploration is provided (Marley, Levin, & Glenberg, 2010). Relating concepts to students' daily life experiences through student-centered pedagogies helps in knowledge construction. According to the constructivist school of thought, students' prior or background knowledge, ideas, and involvement in activities help them in building new concepts (Boudourides, 2003).

Active learning is a shift from traditional teacher-centered methods to student-centered methods and comprises an inclusive and more personalized way of teaching by designing learning environments that uses more visual aids and individualization (Abdullah & Yang, 2019). Active learning strategies follow an organized systematic process. Auster and Wylie (2006) suggested four aspects during active learning in the classroom i.e., context setting, class preparation, class delivery, and continuous progress. Context setting means creating an open and favorable learning environment for students. Class preparation refers to rigorous planning and preparation by the teacher before the class. Class delivery means the execution of the planned lesson and continuous progress comprises collecting and utilizing feedback to improve instructional strategies. Common active learning strategies are; concept-tests, thinking-aloud pair problem solving (TAPPS), one-minute paper, fishbowl, think/share/pair, buzz groups, role-playing, debates, jigsaw group project, roundtable, brainstorming, and mind mapping (Kerrigan, 2017; Roehl, Reddy, & Shannon, 2013).

Active learning accelerates students learning by enabling them to organize, interpret, and connect the new concepts, knowledge, and skills into their intellectual schema (Stanberry, 2018). Active learning strategies in comparison with traditional teaching methods develop students' higher conceptual knowledge and promote self-regulated learning (Kerrigan, 2017). Active learning strategies develop students' interests, their curiosities, encourage their participation, and consequently increase students' academic achievement as compared to the traditional classrooms (Corkin et al., 2017, Edoh, Kurepa, & Roop, 2017, Eddy & Hogan, 2014). Moreover, learners feel excited when they are engaged in the active learning process and completing tasks with their peers under the supervision of the teacher (Kramer, Brewé, & O'Brien, 2008). Learners improve in cognitive, affective, and psychomotor domains during active learning that enhances their decision-making skills (Freeman et al., 2014). Students self-regulate their learning if they are given opportunities to make decisions regarding the various aspects of the learning process; hence, make students responsible for their learning (Clark, Stabryla, & Gilbertson, 2018; Zimmerman, 2015). According to Abdullah and Yang (2019), Davidson (2016), Lugosi and Uribe (2020), and Suherman et al. (2011), active learning significantly contributes to students' academic achievement, particularly in mathematics.

To use active learning in the classroom, the arrangement of active learning strategies in proper sequence is needed. This arrangement may be in the form of a learning model. According to Gagne', Wager, Goals, and Keller (2005), the instructional model is the systematic development of instructional specifications using learning and instructional theory to ensure the quality of instruction. The learning model includes the development and organization of content, instructional strategies, and evaluation process. A variety of instructional models that exist for providing instructions in an organized manner are available for active learning in mathematics. A brief overview has been given below:

Active Learning Model for Classroom Management; Whittington and Yacci developed this model in 2008. It consists of four phases i.e. what (Lecture), how (in-class activities), where (Homework assignments), and why (Reflective group activities).

Mathematical Learning Model; Knisley developed Mathematical Learning Model in 2000. This model consists of four steps e.g., allegorization, integration, analysis, and synthesis.

BSCS 5E Instructional Model; BSCS 5E Instructional Model was developed by Bybee in 1997. This model has five steps like engagement, exploration, explanation, elaboration, and evaluation.

Gagne's (1985) Nine Events of Instruction; These nine events are gaining attention, informing learners about objectives, stimulating recall of prior learning, presenting the content, providing

learning guidance, eliciting performance, providing feedback, assessing performance, and enhancing retention.

Instructional Delivery for Active Learning Framework; Beattie and Rhoads developed the Instructional Delivery for Active Learning Framework in 2005. This model completes in five steps i.e., activation, presentation, application, reflection, and students' guidance and support.

Abdullah and Yang (2019) identified four key variables that affect students' achievement in mathematics are motivational values, interests, attitudes, and self-confidence. The motivational values variable relates to the students' motivation in solving the mathematical problems. The second variable interest is concerned with students' enjoyment in solving the mathematics questions. Attitude as the third variable relates to students' positive and negative attitudes towards mathematics tasks. Self-confidence describes students' willingness and readiness to solve mathematical problems.

In Pakistan, the government is taking serious measures in increasing the literacy rate along with the quality of education. Active learning may help students in learning mathematics by doing tasks and involving in problem-solving activities. In this scenario, the study was conducted and aimed to develop and implement the model for finding its validity at the secondary level in terms of academic achievement in the subject of Mathematics.

Research Methodology

This study was conducted in two phases including the development of the proposed model and testing of the model. The first phase contains development, validation, and description of the model; and the second phase is testing the model through an experiment.

Phase 1: Development of 4WsHs model of active learning for teaching mathematics

The researcher thoroughly reviewed related literature and already existing different learning models before developing the purposed model. Although the prevalent active learning models are very worthwhile due to their practical application, these are not developed specifically for teaching mathematics through active learning. As noted by Doosti and Ashtiani (2005), a mathematical model or any active learning approach takes more time for making a decision about the selection of classroom activity than the traditional approaches. To overcome this constraint the researchers were interested in developing a new model of teaching mathematics through active learning. The researchers developed a new proposed active learning model based on a thorough study of related literature and other active learning models developed by different experts. The newly proposed model contained four phases that were aligned with four stages suggested by Knislay (2000) in the mathematical learning model. It is also concordant with Gagne's (1985) nine events of instruction.

Validation of the proposed model

The researchers validated the newly developed model using the Delphi technique (as suggested by Skulmoski, Hartman, & Krahn, 2007) with three rounds.

1. Round 1

The initial draft of the proposed active learning model for teaching mathematics at the secondary level was comprised of six phases including what (lecture), how (in-class activities presented), Where (homework assignment assessment), why (reflective group discussion and feedback). A validation checklist along with the draft of the developing model was presented to the committee of experts. The experts provided useful suggestions for the improvement of the model (Table 1). The researchers analyzed these suggestions and improved the model accordingly.

Table 1:

Experts Opinions for Phase-1 about the Validation of Proposed Active Learning Model

S No.	Factors	Fully	Partial	Not at all	Recommendations
1	Calibration	5	2	1	The model is not flexible.
2	Face validity	6	1	1	Phases are not clear.
3	Theory validation	4	2	2	What theory does this model follow?
4	Veridicality	3	3	1	How this model helps in making decisions about learning outcomes?
5	Agent behavior validation	5	3	0	Teachers' role is not defined
6	Validation of emergent	4	3	1	How learning be fun through this model?
7	Structure and process	6	1	1	What activities does the teacher follow in a specific phase?

Total	32	15	7
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Table 1 shows that experts were partially satisfied with the validity factors of the proposed model.

2. Round 2

Modifications were made in the division of the draft model into four phases as suggested by experts. Now the proposed model includes 4 phases; what and how (activation and presentation), where and how (application), why and how (reflection), and were on and how (assessment and feedback). The improved copy of the model was again presented to the experts with the same validation checklist. It was recommended by the committee that phases of the model may be clearer and some active learning strategies specific to the mathematical concept should be added. In this round of validation, the committee of experts suggested a few changes with positive critique (Table 2). These suggestions by experts were incorporated to design and finalize the 4Ws4Hs active learning model specific for teaching mathematic at the secondary level.

Table 2:

Experts Opinions for Phase 2 about the Validation of Proposed Active Learning Model

S No.	Factors	Fully	Partial	Not at all	Recommendations
1	Calibration	7	1	0	The model should be flexible following mastery of a concept.
2	Face validity	8	0	0	
3	Theory validation	8	0	0	
4	Veridicality	7	1	0	Specific active learning activities for each according to the mathematical concept.
5	Agent behavior validation	8	0	0	
6	Validation of emergent	7	0	1	Teachers' role for each phase of a proposed model like a motivator, presenter, guide, coach, and judge.
7	Structure and process	7	1	0	
	Total	52	03	01	

Mean= 52/56=0.982

Table 2 shows that the majority of experts regarded the newly developed model as valid concerning factors of validity. Overall, 98.2 % of the experts depicted satisfaction with the validity of the model.

3. Round 3

In this round, the researchers again presented the draft model to the experts after incorporating the recommendations of step 2. This draft contained four phases as discussed in phase2 and this time active learning activities according to the nature of mathematical concepts were also added following the recommendations of a committee of experts. Moreover, each phase was further described. This interactive model 4Ws4Hs was constructed in its final form. The more facilitating aspect of the model was that it had guidelines about deciding the choice of more suitable active learning strategies for different concepts of mathematics. The proposed model of active learning for teaching mathematic is denoted by 4Ws4Hs; 4W stands for 'what', and these indicate 4 steps to be taken, and 4H stands for 'how' which shows an implementation of specific 4 steps in the classroom.

Fig. 1: 4Ws 4Hs Active Learning Model for Teaching Mathematics at Secondary Level

Description of the Model

The four different phases of the model have been explained as under:

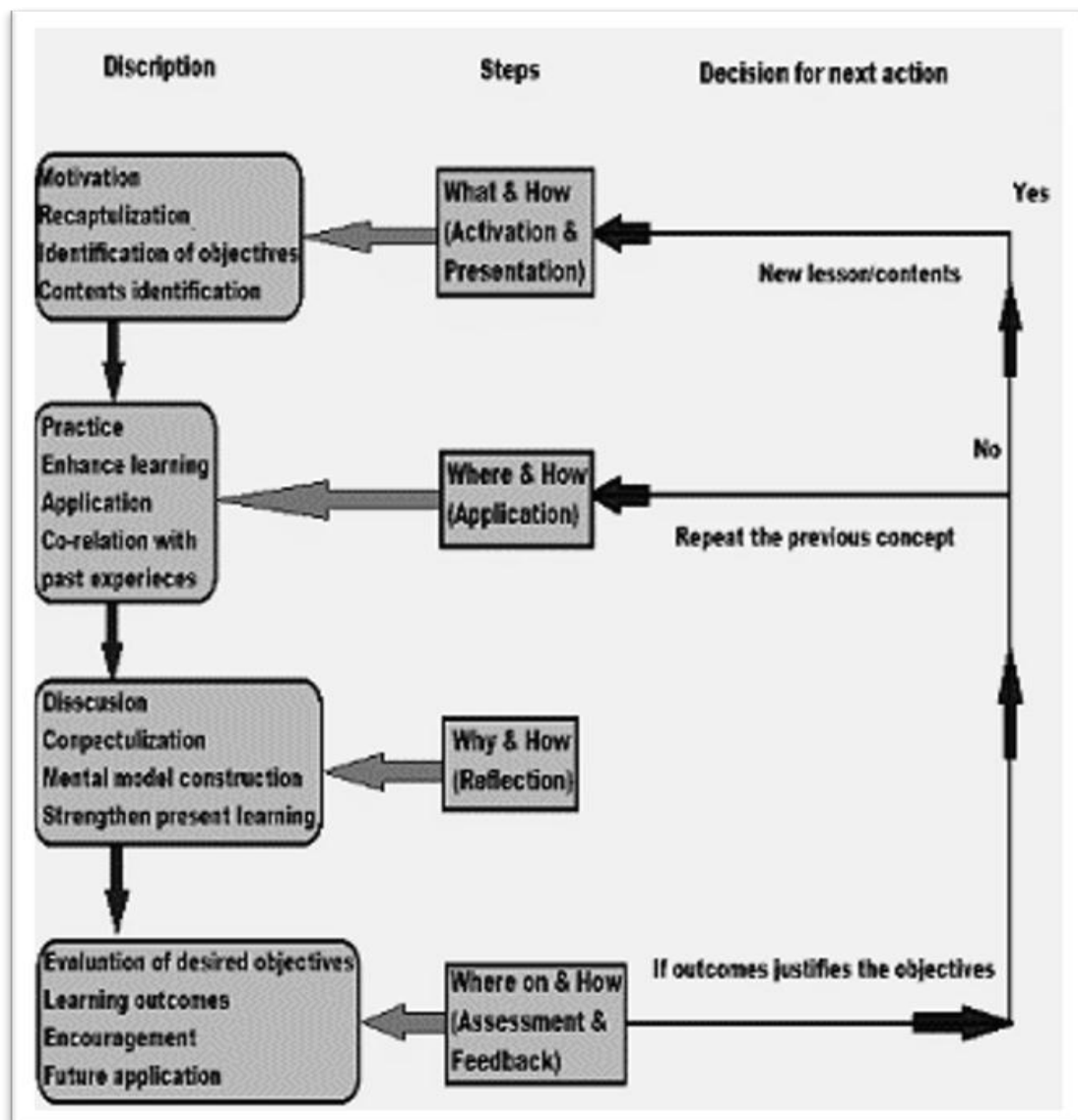
1. What & How (Activation & Presentation)

This phase has two stages: activation (what) and background knowledge prob. Activation (what) as the first stage is meant to prepare the student to learn new concepts and find out their prior knowledge about the concept under consideration. At this stage, the teacher motivates the students for learning on a priority basis. This stage is aligned with the first three events from 'Nine events of instruction design by Gagne (1985).

Suggested Active Learning strategies

- Brainstorming Mind-mapping
- Show off hands Showdown
- Thumbs all together Let's share

- Pros & cons Questions
- Background Knowledge Prob



The second stage of phase-1 consists of presenting new learning materials and selecting activities suitable for the concepts to be taught in the classroom, students need the presentation of information in a suitable form by the teacher. Book reading or lectures are the different ways to present information to the students. The fourth and fifth events of Gagne's events of instruction are addressed through these materials. A teacher plays the role of a storyteller at this stage when he/she delivers a lecture to present content-related information to the learners.

Suggested Active Learning Activities

- Brainstorming Asking Question
- Head together Interactive handout
- Jigsaw Guided-discovery

2. Where & How (Application)

The second phase of the model offers the activities to practice and apply the learned concept to strengthen the learning of students. This phase of the application is directly related to learning outcomes. This phase of the model addresses the sixth stage of Gagne's (1985) nine events of instruction, and at the same time, it is aligned with the second stage (integration) of the mathematical model of active learning where despite having the realization of the concept, the learner is unable to relate it to the known concepts. Therefore, at this stage, the students have to integrate their existing knowledge with the new concept and accommodate in a form of a new idea. This process helps in

developing a strong base for learning new ideas. For example, students learn to solve numerical problems related to a target concept. The role of the teacher in this phase is to act as a guide or motivator. There may be different ways to carry out the application phase, and it is not limited to a specific activity; developing a presentation or project work, group work or individual activities may be used in the application phase.

Suggested Active Learning Activities

- Fish-bowl concept/mind map
- Send a problem Round table
- Jigsaw collective problem
- Problem solving visible question
- Pair check pass a problem

3. Why & How (Reflection)

This phase helps students in articulating concepts and building up mental models for a letter use. This phase is aligned with the seventh event of Gagne's (1985) nine events of instruction. Cooperative learning activities in small groups are suitable for solving problems in this phase. It is an open-ended phase in which students have to just write or discuss their experience or describe what they have learned. This phase also helps in enabling students to build a large content for why the learning experiences or materials that they have been provided are so valuable or important. It develops a linkage between learning materials and further learning. A teacher at this stage being a source of information is primarily a facilitator of discussions and has three main responsibilities in this regard i.e. arrange discussions, initiate the discussion by posing questions; and conclude the discussion with significant reflection.

Suggested Active Learning Activities

- Windows of Wisdom One Minute paper
- Reciprocal Learning Think-Pair-Share
- Activity Matrix Class Discussion
- Focused Freewriting Metacognition
- Matching Questioning

4. Where on & How (Assessment & Feedback)

In the final phase, the model caters to the assessment of students learning and provides feedback about weak areas of learning. It is helpful for students in identifying and improving deficient areas and using information for improving future learning and performance. These supporting aspects of the model help accommodate new concepts with unique identities and hence prove to be a source of skill development (Kinsley, 2002). The last two events of Gagne's (1985) nine events of instruction are covered in this phase of the proposed model. It also deals with summative and formative assessment of students in the learning process of students (Bybee, 1997). The basic element of the feedback must include scaffolding (tips, suggestions, and hints) and constructive encouragement that is supportive of the learning and development of concepts. It requires free forward feedback and helps in developing among students confidence for accuracy and problem-solving. For this purpose, the teacher needs to solve a problem as a model, then make students groups solve problems, and lastly give a problem for the solution for each individual. At this stage, mastery level is achieved by students and they can solve other similar problems. Furthermore, they develop concrete allegories and problem-solving strategies. The teacher act just like a coach in this final stage.

Suggested Active Learning Activities

- Tree branching Flash card
- Name that terms One-minute paper
- Background Asking questions
- Problem solving Think-pair-share
- Three step interview Pair check

Phase 2: Testing of 4Ws & 4Hs Model of Active Learning for Teaching Mathematics

For testing the developed model, a pre-test equivalent grouse design was used as a research method for experimentation. This research design helps in equating the experimental group and control and ensures the elimination of threats to internal validity (Fraenkel, Wallen, & Hyun, 2012).

Population and Sample

The population of this study was comprised of all the students of secondary level classes of the two schools selected for the experiment. The total population for this study was 308 students of both Government high Schools Mansehra. From both schools, a sample of 190 was selected as a sample. For this selection, matched random sampling technique was used.

Research Instrument

To test the effectiveness of the newly developed model, the research instrument was a teacher-made academic achievement test that consisted of 100 items with four multiple choices. The academic achievement test was constructed from Unit-II, unit-III, unit- IV, and unit IX of the 9th class mathematics textbook. For pilot testing, the academic achievement test was administered to 40 students taken from the 9th class of GHSS Sherpur Mansehra. This testing reflected a few problems regarding statements of questions and the difficulty level of the items. The discrepancies were removed, and language was made clearer for understanding. The test was properly validated through expert opinion, and its reliability coefficient was calculated using the Spearman-Brown Prophecy formula, and its value was found to be 0.85.

Treatment Procedure

For forming experimental and control groups, a pre-test was conducted. The students were divided into two equated groups using matched allocation of students to these groups. The experiment was conducted in two Government High schools of district Mansehra. The material selected for the experiment was arranged in 30 lessons plans, following the newly developed active learning model 4Ws4Hs for teaching mathematics. Both experimental and control groups in one school were taught by the researcher (first author) while the both experimental and control groups in the second school were taught by another teacher having experience and qualification same as an experimenter of the first school. Teachers of both schools were provided training for one week about the implantation of the 4Ws4Hs active learning model. The researchers regularly visited the school for providing a guideline and having feedback and resolving the problems (if any). Treatment was provided for two months. Both experimental and control groups covered the same learning materials during experimentation, but the traditional method of teaching was used in the classroom of the control group. Again, at the end of treatment, the achievement test of mathematics was conducted as a post-test to find the comparative achievement of experimental and control groups.

Data Analysis

Statistical tests like mean achievement scores, standard deviation, and effect size were used for the process of data analysis. The analyzed data provided a base for determining findings, drawing a conclusion, and making recommendations.

Results

The mean academic achievement score of students of both control and experimental groups was found out on Pretest.

Table 1:

Mean Academic Achievement Scores of Experimental and Control Groups on Pre-test

No.	Comparison groups	No. of students	Mean scores	S.D	d	R
1	Experimental group	95	27.70	11.43	0.009	0.0004
2	Control group	95	27.69	11.33		

Table 1 (d=0.009, r=0.0004) indicates no significant difference between the mean academic achievement scores of experimental and control groups on pre-test that depicts the same academic achievement level of both groups before treatment.

Table 2:

Mean Academic Achievement Scores of Experimental and Control Groups on Post-Test

No.	Comparison groups	No. of students	Mean scores	S.D	D	R
1	Experimental group	95	46.35	12.25	0.85	0.388
2	Control group	95	36.12	11.69		

Table 2 (d=0.85, r=0.00) indicates a significant difference between the mean academic achievement scores of experimental and control groups on post-test. The mean academic achievement score of the experimental group (46.35) exhibits better performance in the subject of mathematics than the academic performance of the control group (36.12) on post-test.

Discussion

The results depicted that students were at the same academic achievement level in mathematics on a pre-test before receiving any treatment; however, the findings on the post-test revealed a significant difference in academic performance in mathematics after teaching from the new model. The findings of the current study are consistent with the findings of the studies conducted by Lugosi and Uribe (2020), Abdullah and Yang (2019), Corkin et al., (2017), Edoh, Kurepa, and Roop (2017), Davidson (2016) and Suherman et al., (2011). They found out that active learning significantly contributes to students' academic achievement in mathematics. The findings of the current study signify that the mean academic achievement score of the experimental group taught by using active learning is better than the control group that is taught using traditional methodologies. These findings corroborate with the findings of the study conducted by Roop et al. (2018) and Afzal, Gondal, and Fatima (2014). They found that the students using active learning in the Student-Centered Activities for Large Enrollment Undergraduate Programs (SCALE-UP) sections performed better and the average course grade was 0.27 points higher with a significance level of $p = 0.01$ than the sections taught with using traditional methodology. The findings of the study are also consistent with the research study conducted by Afzal et al. (2014). They found the impact of three instructional methods, i.e. active learning, traditional instructional method, and computer-assisted instruction reported better performance of the students taught by using active learning. The results of the study are also supported by Ball and Bass (2003) found out the inquiry method is the best active learning strategy to boost the academic achievement of students.

The current study demonstrated a statistically significant increase in students' academic performance based upon their active engagement and time spent in active learning in mathematics. These results are coinciding with the findings of the previous study conducted by Ting, Lam, and Shroff (2019), who depicted a significant difference in mathematics achievement when taught through active learning strategies. Similarly, the results of the current study are following the studies by Sidhu & Srinivasan (2018) and Theobald et al. (2020), who found out that students taught using the instructions based on the active learning strategy performed significantly better in the mathematics tests than the students taught via traditional methods.

Conclusions

It was concluded that for making learning an active and dynamic process, teachers need to play his/her role as guides and facilitators. The proposed 4Ws4Hs active learning model for teaching mathematics has empirically proved to be a very suitable method of teaching for the subject of mathematics. This model helps in achieving the desired learning goals of education as these have been set at the secondary school level. This model ensures the active participation of the student in learning mathematics. It also provides the activities for the learners that are complementary for achieving the desired learning goals of education as well mathematics at the secondary school level.

On the whole, 4Ws4Hs active model of a learning model for mathematics has proved itself comparatively as an effective teaching-learning strategy. It is because of its active learning environment where students more actively take part in solving mathematics problems as compared to traditional teaching methods. The findings of this study have implications for mathematics teachers teaching at the school level. As 4Ws4Hs active learning model for teaching mathematics has proved to be effective for teaching mathematics at the secondary level, therefore, it may be better to test and apply at other levels (elementary for high) for teaching mathematics, as well as other subjects.

To cope with the challenges of teaching mathematics, training is arranged for mathematics teachers for applying this model in the mathematics classes. For ensuring the application of this model in teaching mathematics, sufficient monitoring and feedback may be used. In the experimentation of the current study, the students had to change their seating arrangement to participate in different activities of active learning, and it was time-consuming. Therefore, the researchers in future studies are recommended to use a flexible seating arrangement where students can form arranged activities for active learning.

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