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Normality Testing in Medical Data: Classical vs. Adapted Jarque-Bera

* Syed Wajahat Ali Bokhari

** Nasir Ali

Abstract

The principle of normality stands as a crucial component for statistical procedure specifically in medical research since parametric methods such as t-tests ANOVA and regression analysis depend on normally distributed data for proper inference. Real medical data databases deviate from normal distribution patterns when researchers observe skews and outliers or heavy-tailed distributions because such variations negatively impact statistical research outcomes. A performance evaluation examines classical and modified normality tests for medical data distribution assessment. Two proposed adaptations known as First Adapted Jarque-Bera (JB) alongside Second Adapted Jarque-Bera (JB**) join a comparison with traditional tests that include Shapiro-Wilk, Anderson-Darling, Kolmogorov-Smirnov and Jarque-Bera and Lilliefors and Shapiro-Francia tests and D'Agostino's skewness and Anscombe-Glynn kurtosis and Pearson Chi-Square Test results alongside* JB_a, $JB_{\sigma 2}$, $JB_{a,\sigma 2}$ *tests. The study carries out a detailed simulation analysis with 20,000 replicates among different sample sizes (10, 20, 30, 50, 100) to evaluate these tests against symmetric and asymmetric distributions. Standard testing techniques demonstrate good performance at large sample levels but show limitations when dealing with small sample-based and non-normal data which causes erroneous type I or type II detection rates. The JB* and JB** tests prove to be robust since they sustain stronger error control together with superior detection capabilities in multiple distributional environments. A correct assessment of normality remains essential for valid statistical inference in clinical trials together with biomedical studies. The presented study enhances medical statistics methodology by developing reliable statistical analyses for healthcare decisions making.*

Keywords: Normality tests, Shapiro-Wilk test, Anderson-Darling test, Kolmogorov-Smirnov test, Jarque-Bera test, Bootstrap method, Robust estimators, Medical data analysis, Statistical inference, Simulation study.

Introduction

The assumption of normality lies at the foundation of statistical analyses because it enables valid use of parametric methods including t-tests along with ANOVA and regression analysis and confidence interval estimation. These statistical procedures deliver valid p-values as well as confidence intervals and effect sizes because they require normal distribution data Altman (1991). Positive or negative skewness and heavy-tailed distributions together with outliers appear frequently in real-world datasets which deteriorate statistical analysis inferences particularly when sample sizes remain small Royston (1992).

Medical research relies on normality distribution to conduct reliable statistical analysis in both clinical trials and epidemiological studies as well as biomedical investigations according to Rosner (2015). The application of statistical modeling requires the assumption that blood pressure readings along with cholesterol levels and patient biometrics should follow a normal distribution per Bland and Altman (2000). Medical data irregularities have a major effect on hypothesis testing and predictive modeling together with risk factor assessment resulting in incorrect clinical choices Ghasemi and Zahediasl (2012). Research validity depends on normality evaluation which produces reliable and robust medical research findings.

Various statistical tests assess normality, including the Shapiro-Wilk (SW) test by Shapiro and Wilk (1965), Anderson-Darling (AD) test by Anderson and Darling (1952), Kolmogorov-Smirnov (KS) test by Kolmogorov (1933); Smirnov (1948), Jarque-Bera (JB) test by Jarque and Bera (1980), Lilliefors (LL) test by Lilliefors (1967), Shapiro-Francia (SF) test developed by Shapiro and Francia (1972), D'Agostino's skewness (S) test by D'Agostino (1970), Anscombe-Glynn kurtosis (K) test by

* Department of Statistics, PMAS-Arid Agriculture University, Rawalpindi, Pakistan
Email: Wajahatbokhari2@gmail.com

** Department of Statistics, PMAS-Arid Agriculture University, Rawalpindi, Pakistan

Anscombe and Glynn (1983), Pearson Chi-Square (χ^2) test by Pearson (1900), and modified Jarque-Bera (JB_a, $JB_{\sigma}2$, $JB_{a,\sigma}2$) tests by Glinskiy et al. (2024). The sensitivity levels of these tests vary for detecting tail deviations and skewness and kurtosis properties because this leads to different test results when describing sample size characteristics.

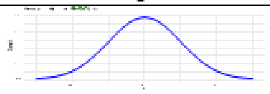
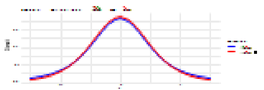
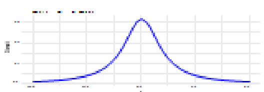
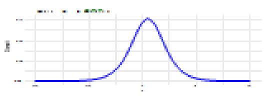
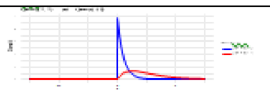
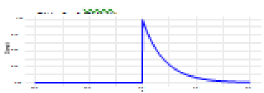
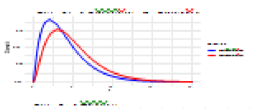
Any assessment strategy focused on verifying normality distributions faces fundamental restrictions in practice. Such tests prove inadequate for small sample sizes resulting in more incorrect negative results for deviations from normality distribution Razali and Wah (2011). The testing methods experience enhanced Type I error rates when dealing with situations where skewness and kurtosis levels become extremely pronounced thus triggering improper normality assumption rejection. Research scientists regularly look for different statistical procedures including robustified normality testing and nonparametric methods to enhance their analysis results.

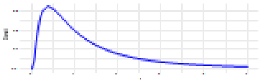
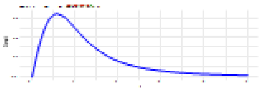
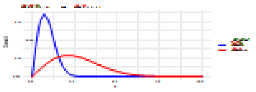
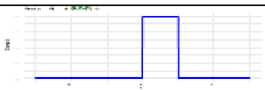
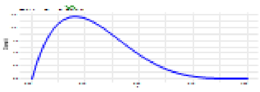
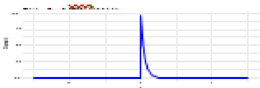
The field of finance together with biomedical and environmental research requires examination of data having heavy-tailed and asymmetric distribution types. The application of classical parametric tests that depend on normality assumptions to such skewed datasets produces misleading outcomes which negatively affects Type I error control and statistical power Kim (2013). The effectiveness of normality tests conducted on distribution types and sample sizes remains unidentified according to research findings which include the Shapiro–Wilk and Jarque–Bera tests Mendes and Pala (2004). Researcher communities have made proposals about modifying testing approaches with robust estimation methods and resampling approaches to enhance accuracy.

The First Adapted Jarque-Bera (JB*) and Second Adapted Jarque-Bera (JB**) tests provide improved approaches for detecting normality by using alternative skewness and kurtosis estimation methods Korkmaz et al. (2014). The JB* test implements median-based Pearson estimators for calculating skewness and uses the median absolute deviation (MAD) to reduce kurtosis estimation sensitivity to extreme values. Nonlinear weighting performed on skewness and kurtosis components through the JB** test increases its capability to find deviations from normality in distributions that are not normal. Both procedures use bootstrapping methods for critical value generation which allows for proper Type I error control and distribution-independent performance at various sample sizes.

The research included a simulation analysis of 20,000 trials for five experimental sample sizes from 10 to 100 and 30 to 100. A wide variety of distributions incorporating symmetric along with asymmetric and light-tailed and heavy-tailed distributions received analysis in the study according to their support and symmetry characteristics. The summary of studied distributions appears in Table 1.

Table 1: Comparison of different distributions classified by support and symmetry. Each cell contains the plot of the respective distribution.

Symmetry	Support	Distributions	Density Plot
Symmetric	$(-\infty, \infty)$	Standard Normal (0,1)	
		Student-t (df=3) and (df=5)	
		Cauchy	
		Logistic(1,2)	
Asymmetric	$(0, \infty)$	Gamma(2, 2) and Gamma(1, 5)	
		Exponential(1)	
		Chi-Square(df=4) and z(df=5)	

		Lognormal(0, 1)	
		Loglogistic(1, 2)	
		Weibull(2) and Weibull(2, 3)	
Asymmetric	0, 1	Uniform(0, 1)	
		Beta(2, 5)	
		Gompertz(10, 0.001)	

The researchers conducted a systematic evaluation of JB* and JB** against standard tests which showed their enhanced ability to check normality in irregular datasets. The developed findings help establish better methods to deal with non-normal data in statistical analysis and specifically support medical research since accurate inference leads to superior clinical decisions.

Methodology: Normality tests and their implementation

A research investigation determines and contrasts different statistical evaluation approaches for measuring normal distribution fit. This work employs established parametric and non-parametric tests together with First Adapted Jarque-Bera (JB*) and Second Adapted Jarque-Bera (JB**) tests which were recently developed as part of the study. Each test's characteristics and implementation information appears in Table 2.

Table 2: Summary of Normality Tests

Test	Statistic Definition	R Implementation
Shapiro-Wilk	$SW = \frac{(\sum_{i=1}^{n} a_i X_{(i)})^2}{\sum_{i=1}^{n}(X_i-\bar{X})^2}$ where: $X_{(i)}$ = ordered sample values, $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$ (sample mean)	shapiro.test(sample)
Anderson-Darling	$AD = -n - \sum_{i=1}^{n} \frac{2i-1}{n} [\ln F(X_{(i)}) + \ln(1 - F(X_{(n+1-i)}))]$ where: $X_{(i)}$ = ordered sample values, $F(X)$ = CDF of hypothesized distribution	nortest::ad.test(sample, "pnorm")
Kolmogorov-Smirnov	$KS = \sup_x \lvert F_n(x) - F(x) \rvert$ where: $F_n(x)$ = empirical CDF, $F(x)$ = reference CDF	ks.test(sample, "pnorm", mean=mean(sample), sd=sd(sample))
Jarque-Bera	$JB = \frac{n}{6}\left(S^2 + \frac{(K-3)^2}{4}\right) \sim \chi^2_2$ where $S = \frac{\frac{1}{n}\sum_{i=1}^{n}(X_i-\bar{X})^3}{(\frac{1}{n}\sum_{i=1}^{n}(X_i-\bar{X})^2)^{3/2}}$ (Skewness), $K = \frac{\frac{1}{n}\sum_{i=1}^{n}(X_i-\bar{X})^4}{(\frac{1}{n}\sum_{i=1}^{n}(X_i-\bar{X})^2)^2}$ (Kurtosis), $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$ (sample mean)	[illegible]
Lilliefors	$LL = \sup_x \lvert F_n(x) - \Phi(x; \hat{\mu}, \hat{\sigma}) \rvert$ where: $F_n(x)$ = empirical CDF, $\Phi(x; \hat{\mu}, \hat{\sigma})$ = normal CDF with estimated mean $\hat{\mu}$ and std. dev. $\hat{\sigma}$	nortest::lillie.test(sample)
Shapiro-Francia	$SF = \frac{(\sum_{i=1}^{n}(X_{(i)}-\bar{X})(m_i-\bar{m}))^2}{\sum_{i=1}^{n}(X_{(i)}-\bar{X})^2 \sum_{i=1}^{n}(m_i-\bar{m})^2}$ where: $X_{(i)}$ = ordered sample values, $\bar{X}$ = sample mean, m_i = expected normal order statistics	nortest::sf.test(sample)

D'Agostino's skewness	$S = \frac{\frac{1}{n}\sum_{i=1}^{n}(X_i-\bar{X})^3}{\left(\frac{1}{n}\sum_{i=1}^{n}(X_i-\bar{X})^2\right)^{3/2}}$ where: $\bar{X}=\frac{1}{n}\sum_{i=1}^{n}X_i$ (sample mean)	moments::agostino.test(sample)
Anscombe-Glynn kurtosis	$K = \frac{\sum_{i=1}^{n}(X_i-\bar{X})^4/n}{\left(\sum_{i=1}^{n}(X_i-\bar{X})^2/n\right)^2}$ where: $\bar{X}=\frac{1}{n}\sum_{i=1}^{n}X_i$ (sample mean)	moments::anscombe.test(sample)
Pearson Chi-Square	$\chi^2 = \sum_{i=1}^{k}\frac{(O_i-E_i)^2}{E_i}$ where: O_i = observed frequency, E_i = expected frequency	nortest::pearson.test(sample)
Modified Jarque-Bera JB_a	$JB_a = n\left(\frac{S_a^2}{6}+\frac{(K_a-3)^2}{24}\right)$ where: $S_a = \frac{\frac{1}{n}\sum_{i=1}^{n}(X_i-a)^3}{\sigma_a^3}$, $K_a = \frac{\frac{1}{n}\sum_{i=1}^{n}(X_i-a)^4}{\sigma_a^4}$, $\sigma_a = \left(\frac{1}{n}\sum_{i=1}^{n}(X_i-a)^2\right)^{1/2}$	Custom implementation
Modified Jarque-Bera JB_σ	$JB_\sigma = n\left(\frac{S_\sigma^2}{6}+\frac{(K_\sigma-3)^2}{24}\right)$ where: $S_\sigma = \frac{\frac{1}{n}\sum_{i=1}^{n}(X_i-\bar{X})^3}{\sigma^3}$, $K_\sigma = \frac{\frac{1}{n}\sum_{i=1}^{n}(X_i-\bar{X})^4}{\sigma^4}$	Custom implementation
Modified Jarque-Bera $JB_{a,\sigma}$	$JB_{a,\sigma} = n\left(\frac{S_{a,\sigma}^2}{6}+\frac{(K_{a,\sigma}-3)^2}{24}\right)$ where: $S_{a,\sigma} = \frac{\frac{1}{n}\sum_{i=1}^{n}(X_i-a)^3}{\sigma^3}$, $K_{a,\sigma} = \frac{\frac{1}{n}\sum_{i=1}^{n}(X_i-a)^4}{\sigma^4}$	Custom implementation
First Adapted Jarque-Bera (JB*)	$JB^* = \frac{n}{6}\left(S_m^2+\frac{K_m^2}{4}\right)$ where: $S_m = \frac{3(\bar{x}-\tilde{x})}{\sigma}$, $K_m = \frac{\cdots}{\sigma^4}$	Custom implementation
Second Adapted Jarque-Bera (JB**)	$JB^{**} = n\left(\frac{\lvert S\rvert^{2+\alpha}}{C_1}+\frac{\lvert K-3\rvert^{1+\beta}}{C_2}\right)$ where: S is skewness, K is kurtosis, α, β, C_1, C_2 are tuning parameters	Custom implementation

Proposed methodology

Researchers introduce the proposed normality tests JB* and JB** together with their theoretical framework and methodology. The proposed tests seek to boost normality assessment robustness through solutions for Jarque-Bera (JB) test weaknesses.

First adapted Jarque-Bera (JB)**

As a popular statistical method for detecting normality the Jarque-Bera (JB) test depends on the measurement of sample skewness combined with kurtosis. Small sample sets together with non-normal distributions lead to high sensitivity of this test technique towards outliers. The JB* test applies alternative robust measures to skewness and kurtosis assessment by utilizing Pearson's median skewness along with median absolute deviation (MAD)-based kurtosis.

The measurement of distribution asymmetry is called skewness. The regular method used to determine distribution skewness gets easily affected by rarely occurring extreme data points. The robustness enhancement of JB* derives from Pearson's median skewness which defines as:

$$S_m = \frac{3(\bar{x}-\tilde{x})}{s}$$

This scheme uses three statistical values for analysis including the sample mean $\bar{x}$ and sample median $\tilde{x}$ together with the sample standard deviation s. The new formulation maintains the capability to detect data asymmetry without allowing outliers to influence the results.

Kurtosis measures the heaviness of a distribution's tails. The conventional fourth-moment kurtosis is highly sensitive to extreme values. To counter this, JB* introduces a *median absolute deviation (MAD)-based kurtosis*, given by:

$$K_m = \frac{\frac{1}{n}\sum_{i=1}^{n}\lvert X_i - \tilde{x}\rvert^4}{(MAD^4)} - 3$$

where MAD is computed as:

$$MAD = \text{median}(\lvert X_i - \tilde{x}\rvert)$$

The JB* test statistic uses robust measures to form its statistical formulation.

$$JB^* = \frac{n}{6}\left(S_m^2 + \frac{K_m^2}{4}\right) \qquad (1)$$

Second adapted Jarque-Bera (JB)**

The JB** test includes parameters α and β that control sensitivity to skewness and kurtosis which helps to enhance both flexibility alongside robustness. The measures for detecting skewness and kurtosis in the classical framework are defined by the two following formulas:

$$S = \frac{\frac{1}{n}\sum_{i=1}^{n}(X_i - \bar{X})^3}{\left(\frac{1}{n}\sum_{i=1}^{n}(X_i - \bar{X})^2\right)^{3/2}}, \quad K = \frac{\frac{1}{n}\sum_{i=1}^{n}(X_i - \bar{X})^4}{\left(\frac{1}{n}\sum_{i=1}^{n}(X_i - \bar{X})^2\right)^2}$$

The JB** test statistic is defined as:

$$\text{JB}^{**} = n\left(\frac{|S|^{1+\alpha}}{C_1} + \frac{|K-3|^{1+\beta}}{C_2}\right) \qquad (2)$$

The test contains two components whose values multiply: C_1 and C_2. Both α and β determine how sensitive the test reacts to changes. In our implementation, we set:

$$\alpha = 0.5, \beta = 1, \quad C_1 = 1.5, \qquad C_2 = 2.5.$$

Bootstrapping procedure

Under the null hypothesis of normality, the distribution of JB^* and JB** does not necessarily follow a chi-square distribution like the original JB test. The JB* and JB** statistic is expected to be small under the null hypothesis $H_0 : X \quad N(\mu, \sigma^2)$, while significant deviations from normality lead to larger values. The critical value for JB* and JB** is determined through a bootstrapping procedure:

1. Generate 1,000 bootstrap samples from $N(\mu, \sigma^2)$ to match the observed sample size.
2. Compute the JB* and JB** statistic for each bootstrap sample.
3. Determine the 95th percentile of the bootstrap JB* and JB** statistics as the critical threshold.
4. Reject H_0 if the observed JB* and JB** statistic exceeds the critical value.

This bootstrap method ensures effective Type I error control and provides realistic distributions for the proposed tests.

Results and discussion

The performance evaluation of proposed JB* and JB** normality tests required a simulation anal- ysis which contained 20,000 replications throughout each distribution and sample size combination. The study evaluated performance using JB* and JB** tests while considering sample sizes from 10 to 100 as well as normal and t(3), t(5), lognormal(0,1), loglogistic(1,2), logistic(1,2), Gamma(2,2), Gamma(1,5), Beta(2,5), Gompertz(10,0.001), Weibull(2), Weibull(2,3), Exponential(1), Cauchy, Uniform(0,1), Chi-Square(df=4), and Chi-Square(df=5) distributions

The evaluation of Type I error rates under normal distribution conditions appeared in Table 3 and graphically shown in Figure 1. With 20,000 simulations we obtained strong and reliable rates to measure. The JB** test successfully regulated its error rates which stayed around the designated 5% level throughout every simulation while maintaining samples from 15 to 200. Name JB* only failed to perform correctly with a sample size of 10 yet it showed reliable outcomes at larger sample sizes. The traditional Jarque-Bera (JB) and modified JB_a tests displayed marked under-rejection at all times with particular strong evidence for sample sizes below 75 observations which implies a conservative error bias. The established tests SW, AD, LL, SF, S and K provided efficient management of Type I error rates. Small sample sizes resulted in elevated types of errors for the Chi-square (χ^2) test.

The empirical power assessment of these tests was performed through 20,000 simulations under different alternative distribution types. Both JB* and JB** demonstrated their ability to detect deviation from normality in heavy-tailed distributions by increasing their power with sample size based on the results shown in Table 4 and visualled in Figure 2. The power levels were generally highest for $JB_{\sigma}2$ and $JB_{a,\sigma}2$.

The testing power of JB** together with JB* reached high levels for lognormal and loglogistic distributions which demonstrated their potency at detecting skewness patterns (Table 5, Table 6) and also graphically display in Figure 3 and Figure 4. Additionally, $JB_{a,\sigma}2$ displayed perfect detection capabilities because of its sensitivity to these distributions.

The power statistics of $JB_{\sigma}2$ and $JB_{a,\sigma}2$ demonstrated superior performance along with an equal level of power as shown in Table 7 and also shown in Figure 5.

The results in Gamma distributions (Table 8 and also display in Figure 6) indicate strong power for JB** and JB* accompanied by robust performance from SW and AD followed by his extraordinary capability of $JB_{a,\sigma}2$ at bigger sample sizes.

Testing of the Beta distribution (Table 9 and also shown in Figure 7) revealed that SW and AD performance metrics demonstrated better power than JB** and JB*.

Within the Gompertz and Exponential distributions (Tables 10 and 12 and also graphically visualised in Figure 8 and Figure 10) JB** and JB* demonstrated similar power levels to SW and AD together with exceptional power from $JB_{a,\sigma}2$.

According to Table 13 also in Figure 11, the Cauchy distribution confirmed that JB** and JB* matched the detection potential of SW and AD while $JB_{\sigma}2$ and $JB_{a,\sigma}2$ offered spotless outlier detection capability.

The power of SW and AD tests exceeded the power of JB** and JB* during the uniform distribution evaluation (Table 14,Figure 12).

JB** along with JB* displayed strong competitive power according to Chi-square distribution results (Table 15, Table 11 and Figure 13 and Figure 9). Detectors $JB_{\sigma}2$ and $JB_{a,\sigma}2$ detected anomalies near-perfectly and SW and AD tested strongly in this distribution group.

The assessment of 20,000 statistical simulations demonstrated an in-depth comparison between JB* and JB** tests alongside classic methods in which they showed both performance pros and cons. Data characteristics and research objectives determine the selection of proper normality tests according to the research findings.

Table 3: Type I error rates of normality tests at a 5% significance level for different sample sizes under the normal distribution

Test	n=10	n=20	n=30	n=50	n=100
JB**	0.0478	0.0475	0.0498	0.0510	0.0528
JB*	0.0741	0.0409	0.0581	0.0503	0.0432
JB_a	0.0063	0.0183	0.0268	0.0317	0.0394
$JB_{\sigma}2$	0.0389	0.0498	0.0498	0.0525	0.0541
$JB_{a,\sigma}2$	0.0620	0.0646	0.0613	0.0592	0.0550
SW	0.0502	0.0509	0.0507	0.0507	0.0488
AD	0.0483	0.0518	0.0495	0.0520	0.0512
KS	0.0417	0.0480	0.0509	0.0534	0.0552
JB	0.0085	0.0252	0.0316	0.0376	0.0427
LL	0.0491	0.0477	0.0480	0.0472	0.0475
SF	0.0534	0.0534	0.0526	0.0521	0.0522
S	0.0529	0.0513	0.0501	0.0485	0.0473
K	0.0401	0.0487	0.0515	0.0529	0.0556
χ^2	0.0703	0.0501	0.0497	0.0510	0.0519

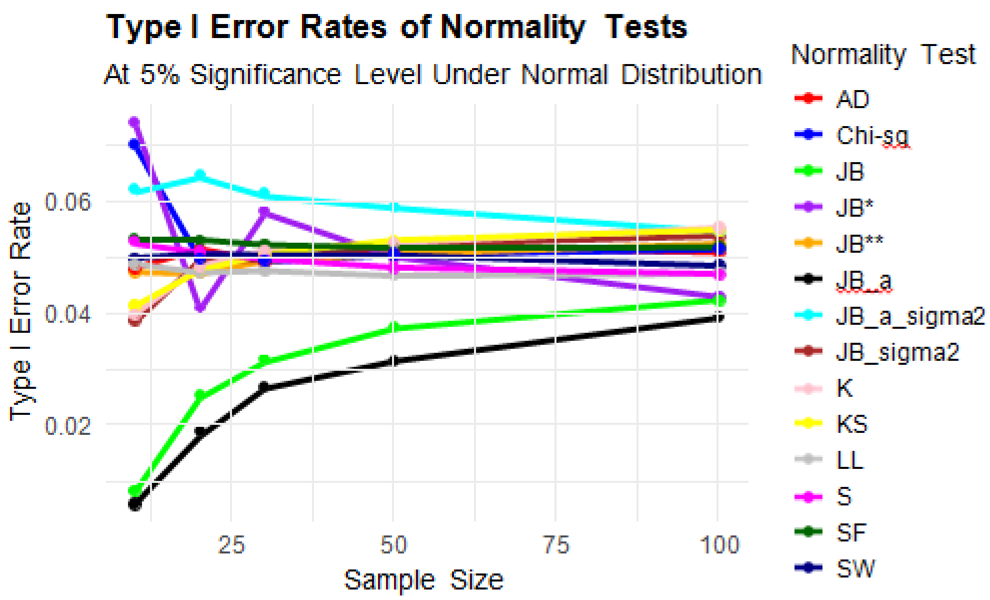


Figure 1: Type I error rates of normality tests at 5% significance level.

Table 4: Empirical power of normality tests under the t(3) and t(5) distributions for various sample sizes

	t(3)					t(5)				
Test	n=10	n=20	n=30	n=50	n=100	n=10	n=20	n=30	n=50	n=100
JB**	0.2230	0.3899	0.5288	0.7116	0.9134	0.1368	0.2347	0.3176	0.4437	0.6707
JB*	0.2303	0.3289	0.5204	0.6916	0.9102	0.1487	0.1651	0.2911	0.3964	0.6235

JB_a	0.1075	0.3169	0.4609	0.6634	0.8945	0.0496	0.1651	0.2521	0.3885	0.6244
JB_{σ^2}	0.4727	0.7041	0.8263	0.9382	0.9950	0.2918	0.4537	0.5732	0.7260	0.9130
JB_{a,σ^2}	0.5394	0.7388	0.8457	0.9434	0.9951	0.3627	0.5026	0.6089	0.7465	0.9199
SW	0.1825	0.3362	0.4588	0.6405	0.87560	0.1117	0.1838	0.2465	0.3579	0.5635
AD	0.1566	0.3083	0.4205	0.5967	0.8505	0.0848	0.1559	0.2097	0.2953	0.4745
KS	0.0060	0.0321	0.0535	0.1087	0.2423	0.0003	0.0034	0.0064	0.0134	0.0267
JB	0.0989	0.3077	0.4617	0.6636	0.8980	0.0453	0.1610	0.2555	0.3998	0.6261
LL	0.1580	0.2529	0.3388	0.4817	0.7385	0.0954	0.1283	0.1546	0.2124	0.3345
SF	0.2109	0.3838	0.5175	0.6973	0.9060	0.1340	0.2179	0.2939	0.4208	0.6309
S	0.2144	0.3420	0.4238	0.5178	0.6381	0.1345	0.2022	0.2519	0.3184	0.4036
K	0.1743	0.3301	0.4627	0.6475	0.8890	0.1044	0.1801	0.2515	0.3747	0.5998
χ^2	0.145[illegible]	0.1767	0.2360	0.3326	0.5324	0.0979	0.0915	0.1050	0.1332	0.1867

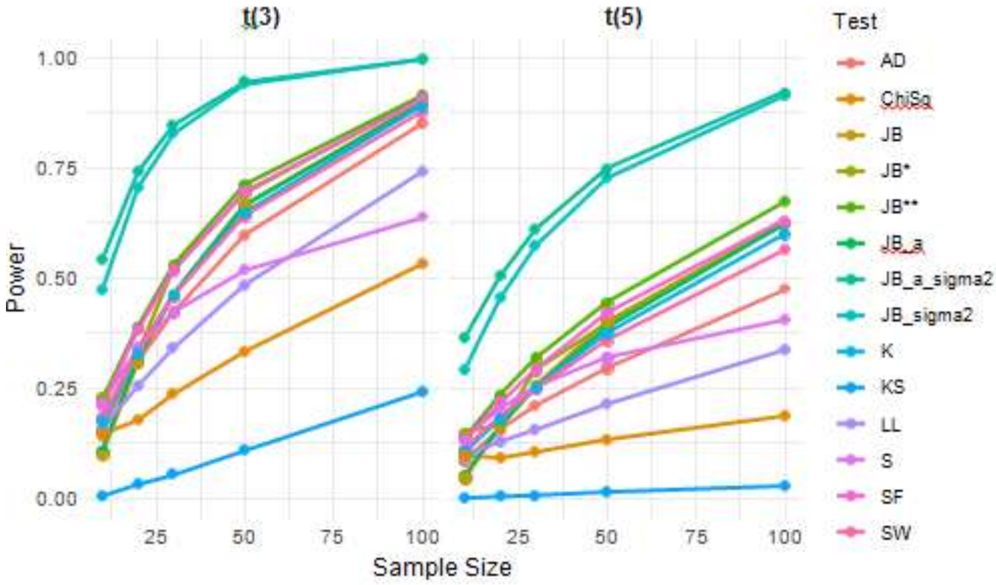


Figure 2: Empirical power of normality tests under t(3) and t(5) distributions.

Table 5: Empirical power of normality tests under the lognormal(0,1) distribution for various sample sizes

Test	n=10	n=20	n=30	n=50	n=100
JB**	0.4603	0.7627	0.9071	0.9909	1.0000
JB*	0.5097	0.7135	0.8908	0.9754	0.9996
JB_a	0.1781	0.5839	0.8701	1.0000	1.0000
JB_{σ^2}	0.5668	0.8225	0.9291	0.9909	1.0000
JB_{a,σ^2}	0.8846	0.9868	0.9992	1.0000	1.0000
SW	0.6041	0.9282	0.9919	0.9999	1.0000
AD	0.5264	0.8953	0.9831	0.9998	1.0000
KS	0.0269	0.1954	0.4316	0.8050	0.9967
JB	0.2838	0.7246	0.9096	0.9958	1.0000
LL	0.4575	0.7871	0.9274	0.9955	1.0000
SF	0.5949	0.9114	0.9873	0.9998	1.0000
S	0.5308	0.8676	0.9683	0.9989	1.0000
K	0.3555	0.5983	0.7525	0.9132	0.9940
χ^2	0.5412	0.8227	0.9491	0.9972	1.0000

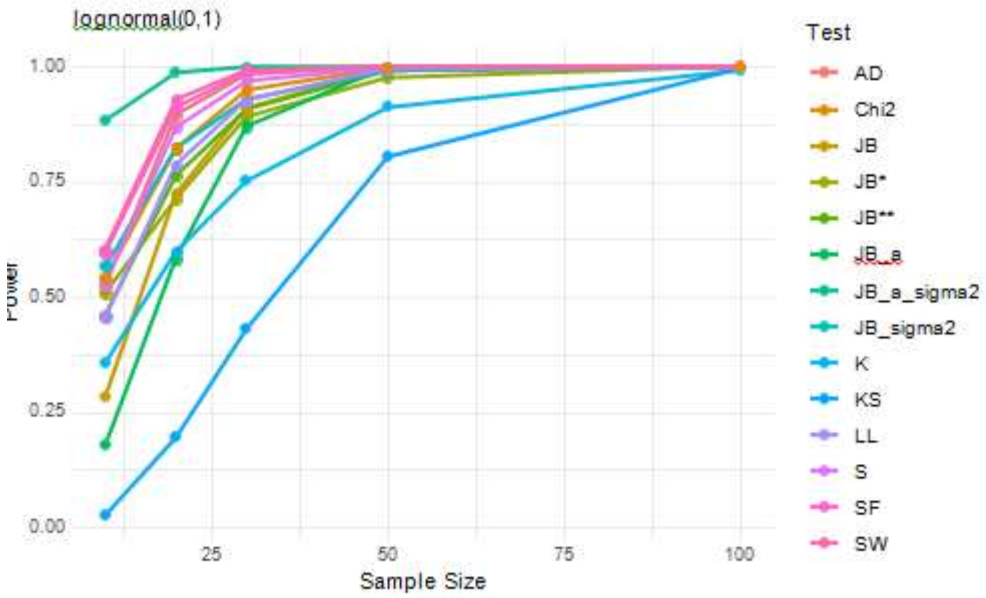


Figure 3: Empirical power of normality tests under lognormal(0,1) distribution.

Table 6: Empirical power of normality tests under the loglogistic(1,2) distribution for various sample sizes

Test	n=10	n=20	n=30	n=50	n=100
JB**	0.4559	0.7574	0.9002	0.9851	1.0000
JB*	0.4708	0.6821	0.8647	0.9649	0.9983
JB_a	0.1723	0.5247	0.7892	1.0000	1.0000
$JB_{\sigma}2$	0.4463	0.6881	0.8252	0.9535	1.0000
$JB_{a,\sigma}2$	1.0000	1.0000	1.0000	1.0000	1.0000
SW	0.5454	0.8735	0.9724	0.9990	1.0000
AD	0.4725	0.8368	0.9569	0.9977	1.0000
KS	0.0429	0.2332	0.4464	0.7656	0.9877
JB	0.2939	0.7129	0.8971	0.9918	1.0000
LL	0.4259	0.7372	0.8883	0.9839	0.9999
SF	0.5476	0.8623	0.9660	0.9983	1.0000
S	0.5156	0.8397	0.9550	0.9977	1.0000
K	0.3641	0.6190	0.7756	0.9242	0.9952
χ^2	0.4709	0.7219	0.8777	0.9819	0.9999

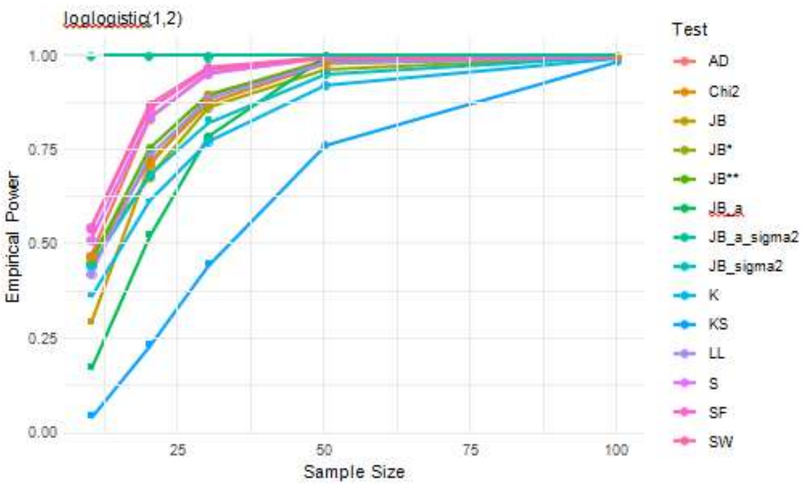


Figure 4: Empirical power of normality tests under loglogistic(1,2) distribution.

Table 7: Empirical power of normality tests under the logistic(1,2) distribution for various sample sizes

Test	n=10	n=20	n=30	n=50	n=100
JB**	0.1014	0.1502	0.1955	0.2723	0.4167
JB*	0.1143	0.1028	0.1830	0.2362	0.3674
JB_a	0.0217	0.0920	0.1542	0.2792	0.5329
$JB_{\sigma}2$	0.9937	1.0000	1.0000	1.0000	1.0000
$JB_{a,\sigma}2$	0.9983	1.0000	1.0000	1.0000	1.0000
SW	0.0807	0.1182	0.1471	0.1963	0.3046
AD	0.4781	0.8312	0.9533	0.9979	1.0000
KS	0.0001	0.0003	0.0008	0.0011	0.0018
JB	0.0261	0.0952	0.1454	0.2243	0.3702
LL	0.0724	0.0815	0.0928	0.1116	0.1599
SF	0.0956	0.1425	0.1822	0.2462	0.3719
S	0.0986	0.1359	0.1574	0.1896	0.2259
K	0.0723	0.1139	0.1435	0.2019	0.3279
χ^2	0.0819	0.0637	0.0702	0.0839	0.0904

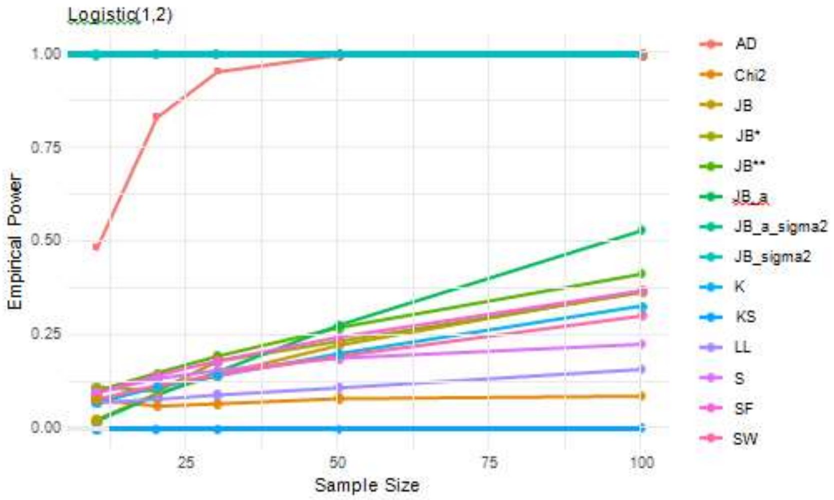


Figure 5: Empirical power of normality tests under logistic(1,2) distribution.

Table 8: Empirical power of normality tests under Gamma(2,2) and Gamma(1,5) distributions for various sample sizes

	Gamma(2,2)					Gamma(1,5)				
Test	n=10	n=20	n=30	n=50	n=100	n=10	n=20	n=30	n=50	n=100
JB**	0.1960	0.3510	0.4956	0.7062	0.9756	0.3007	0.5417	0.7149	0.9230	0.9998
JB*	0.1873	0.2168	0.3699	0.4942	0.7196	0.3287	0.4417	0.6496	0.8090	0.9636
JB_a	0.0112	0.0692	0.2037	1.0000	1.0000	0.0769	0.3608	0.7208	1.0000	1.0000
$JB_{\sigma}2$	0.0364	0.0590	0.0822	0.1296	0.9848	0.0000	0.0000	0.0000	0.0000	1.0000
$JB_{a,\sigma}2$	0.3864	0.6401	0.8292	0.9824	1.0000	0.00005	0.0000	0.0000	0.0000	0.9999
SW	0.2409	0.5335	0.7506	0.9507	0.9997	0.4414	0.8353	0.9672	0.9992	1.0000
AD	0.1724	0.4354	0.6415	0.8852	0.9971	0.3497	0.7466	0.9294	0.9964	1.0000
KS	0.0009	0.0089	0.0214	0.0801	0.3296	0.0043	0.0370	0.1210	0.3772	0.9233
JB	0.0793	0.2859	0.4750	0.7642	0.9908	0.1477	0.4790	0.7201	0.9553	1.0000
LL	0.1689	0.3204	0.4634	0.6910	0.9548	0.3015	0.5750	0.7795	0.9631	1.0000
SF	0.2434	0.5040	0.7075	0.9274	0.9994	0.4306	0.7984	0.9476	0.9982	1.0000
S	0.2295	0.4739	0.6706	0.8935	0.9965	0.4414	0.8353	0.9672	0.9992	1.00000
K	0.1465	0.2299	0.3084	0.4268	0.6383	0.2216	0.3623	0.4789	0.6570	0.8861
χ^2	0.1988	0.2830	0.4074	0.6548	0.9482	0.3914	0.6542	0.8517	0.9837	0.9999

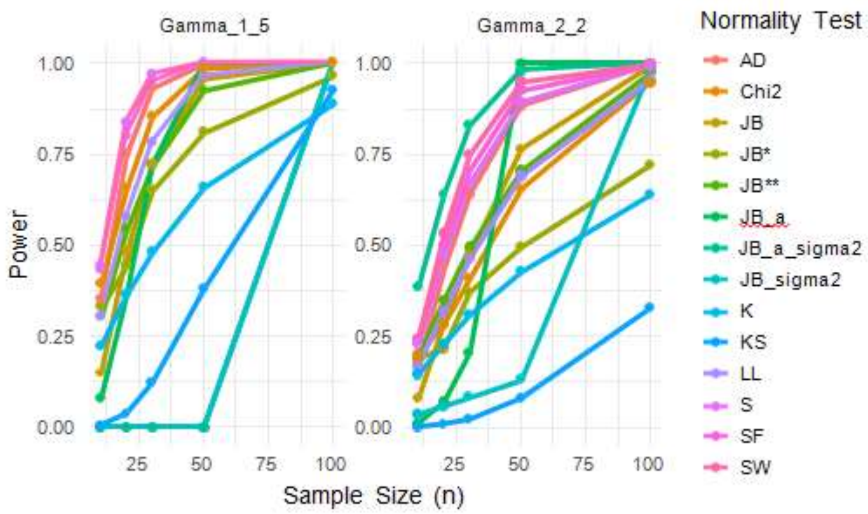


Figure 6: Empirical power of normality tests under Gamma(2,2) and Gamma(1,5) distributions.

Table 9: Empirical power of normality tests under the Beta(2,5) distribution for various sample sizes

Test	n=10	n=20	n=30	n=50	n=100
JB**	0.0652	0.0824	0.0935	0.1429	0.3721
JB*	0.0785	0.0477	0.0669	0.0602	0.0552
JB_a	0.00005	0.0003	0.0013	1.0000	1.0000
$JB_{\sigma}2$	0.0000	0.0000	0.0000	0.0000	1.0000
$JB_{a,\sigma}2$	0.0000	0.0000	0.0000	0.0000	1.0000
SW	0.0889	0.1705	0.2740	0.4975	0.8937
AD	0.0555	0.1324	0.2104	0.3808	0.757
KS	0.0000	0.0004	0.0008	0.0036	0.0185
JB	0.0143	0.0494	0.0789	0.1559	0.5030
LL	0.0728	0.1138	0.1552	0.2575	0.5071
SF	0.0880	0.1448	0.2202	0.4005	0.8249
S	0.0798	0.1300	0.1958	0.3471	0.7027
K	0.0572	0.0787	0.0903	0.1027	0.1129
χ^2		0.091(0.0900	0.1187	0.1912	0.3900

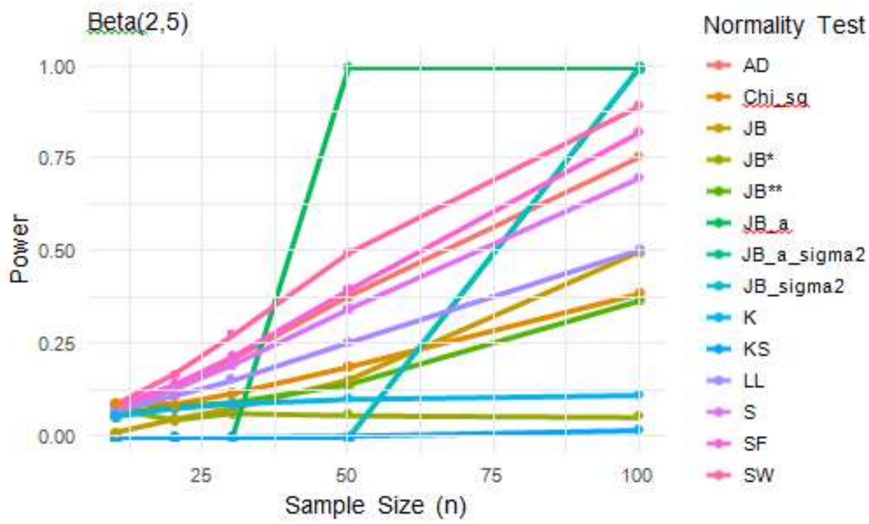


Figure 7: Empirical power of normality tests under Beta(2,5) distribution.

Table 10: Empirical power of normality tests under the Gompertz(10,0.001) distribution for various sample sizes

Test	n=10	n=20	n=30	n=50	n=100
JB**	0.3051	0.5390	0.7212	0.9213	0.9998
JB*	0.3275	0.4446	0.6432	0.8019	0.9609
JB_a	0.0788	0.3608	0.7123	1.0000	1.0000
$JB_{\sigma}2$	0.0000	0.0000	0.0000	0.0000	1.0000
$JB_{a,\sigma}2$	0.0000	0.0000	0.0000	0.0000	1.0000
SW	0.4501	0.8399	0.9665	0.9993	1.0000
AD	0.3524	0.7528	0.9281	0.9969	1.0000
KS	0.0041	0.0396	0.1161	0.3790	0.9211
JB	0.1510	0.4846	0.7233	0.9548	1.0000
LL	0.3001	0.5792	0.7749	0.9597	1.0000
SF	0.4360	0.8020	0.9480	0.9984	1.0000
S	0.3742	0.7056	0.8840	0.9866	1.0000
K	0.2262	0.3700	0.4705	0.6561	0.8825
χ^2	0.4001	0.6590	0.8499	0.9807	1.0000

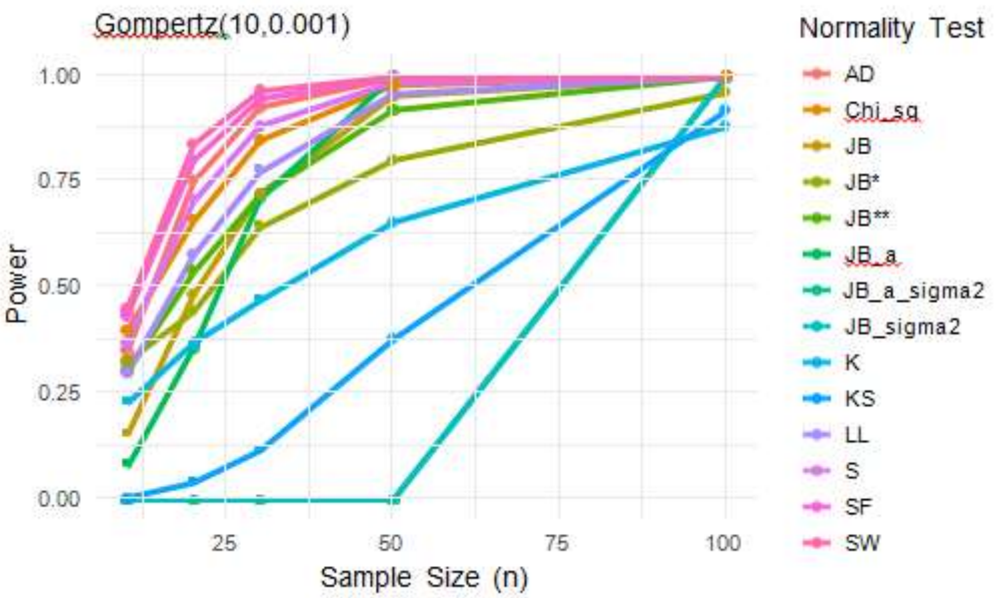


Figure 8: Empirical power of normality tests under Gompertz(10,0.001) distribution.

Table 11: Empirical power of normality tests under Weibull(2) and Weibull(2,3) distributions for various sample sizes

	Weibull(2)					Weibull(2,3)				
Test	n=10	n=20	n=30	n=50	n=100	n=10	n=20	n=30	n=50	n=100
JB**	0.0720	0.1082	0.1375	0.1999	0.3836	0.0760	0.1078	0.1407	0.1961	0.3872
JB*	0.0805	0.0578	0.0876	0.0918	0.1030	0.0870	0.0573	0.0838	0.0933	0.1015
JB_a	0.00005	0.00065	0.0035	1.0000	1.0000	0.00015	0.00055	0.0024	1.0000	1.0000
$JB_{\sigma}2$	0.0000	0.0000	0.0000	0.0000	1.0000	0.3522	0.5720	0.7049	0.8779	0.9880
$JB_{a,\sigma}2$	0.0307	0.0625	0.1436	0.5689	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
SW	0.0867	0.1527	0.2358	0.4124	0.7909	0.0850	0.1525	0.2406	0.4126	0.7936
AD	0.0561	0.1129	0.1739	0.2997	0.6068	0.0562	0.1163	0.1744	0.2989	0.6048
KS	0.00005	0.00045	0.00050	0.00275	0.01135	0.00010	0.00065	0.00115	0.00280	0.01000
JB	0.0175	0.0670	0.1135	0.2105	0.5047	0.0189	0.0659	0.1180	0.2134	0.5060
LL	0.0720	0.0992	0.1350	0.2024	0.3929	0.0728	0.0986	0.1353	0.1999	0.3875
SF	0.0906	0.1416	0.2088	0.3578	0.7185	0.0879	0.1399	0.2126	0.3588	0.7189

S	0.0865	0.1443	0.2165	0.3710	0.6920	0.0838	0.1436	0.2171	0.3712	0.6988
K	0.0635	0.0879	0.1074	0.1304	0.1561	0.0581	0.0850	0.1120	0.1305	0.1539
χ^2	0.0862	0.0768	0.0964	0.1366	0.2564	0.0888	0.0772	0.1020	0.1373	0.2503

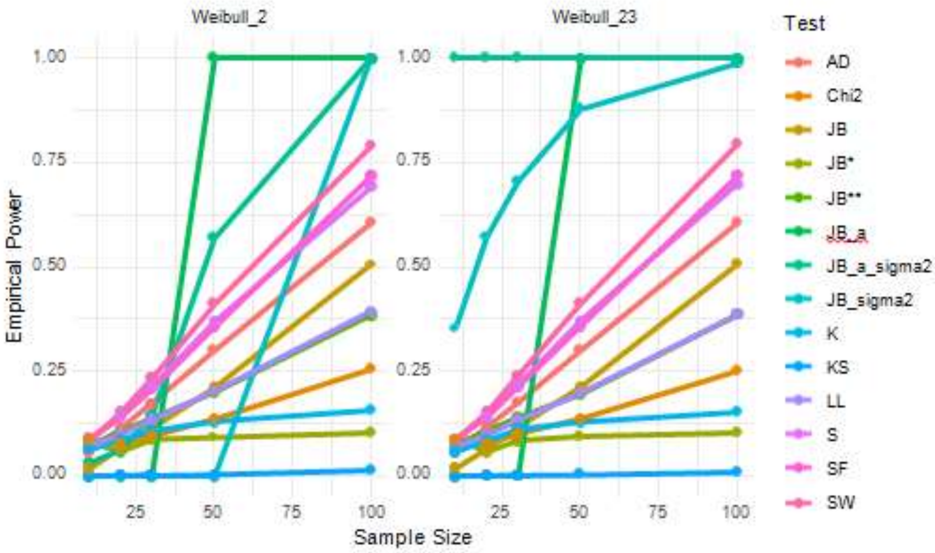


Figure 9: Empirical power of normality tests under weibull(2) and weibull(2,3) distributions.

Table 12: Empirical power of normality tests under the Exponential(1) distribution for various sample sizes

Test	n=10	n=20	n=30	n=50	n=100
JB**	0.3077	0.5450	0.7168	0.9226	0.9995
JB*	0.3250	0.4395	0.6442	0.8038	0.9608
JB_a	0.0754	0.3602	0.7152	1.0000	1.0000
$JB_{\sigma}2$	0.1973	0.3609	0.5001	0.7236	0.9994
$JB_{a,\sigma}2$	0.5818	0.8219	0.9403	0.9942	1.0000
SW	0.4454	0.8359	0.9690	0.9993	1.0000
AD	0.3488	0.7514	0.9270	0.9966	1.0000
KS	0.0049	0.0374	0.1142	0.3767	0.9175
JB	0.1481	0.4781	0.7312	0.9526	1.0000
LL	0.3003	0.5773	0.7785	0.9600	1.0000
SF	0.4335	0.7975	0.9516	0.9980	1.0000
S	0.3712	0.7038	0.8881	0.9866	1.0000
K	0.2234	0.3588	0.4814	0.6551	0.8824
χ^2	0.3916	0.6548	0.8550	0.9828	1.0000

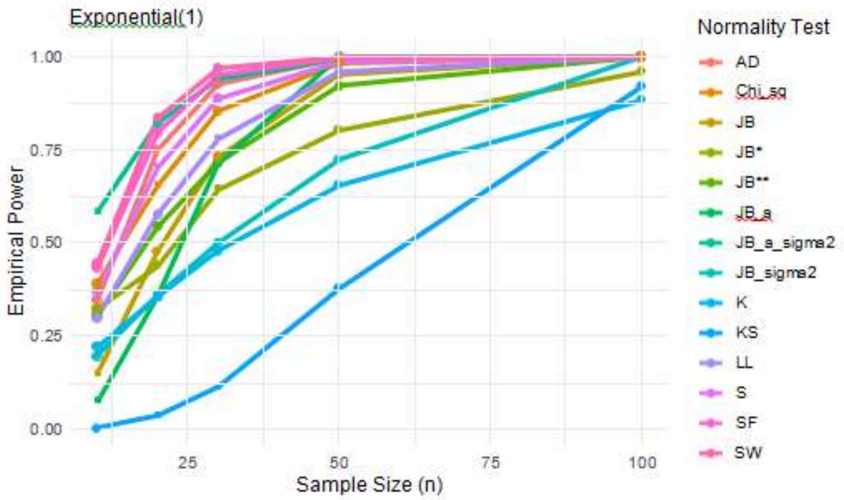


Figure 10: Empirical power of normality tests under Exponential(1) distribution.

Table 13: Empirical power of normality tests under the Cauchy distribution for various sample sizes

Test	n=10	n=20	n=30	n=50	n=100
JB**	0.5913	0.8660	0.9584	0.9974	0.9995
JB*	0.6663	0.8859	0.9768	0.9982	1.0000
JB_a	0.4519	0.8204	0.9450	0.9943	1.0000
$JB_{\sigma}2$	0.9048	0.9905	0.9992	1.0000	1.0000
$JB_{a,\sigma}2$	0.9232	0.9925	0.9993	1.0000	1.0000
SW	0.5834	0.8659	0.9578	0.9973	1.0000
AD	0.5730	0.8757	0.9617	0.9973	1.0000
KS	0.1661	0.5112	0.7315	0.9286	0.9986
JB	0.4181	0.8143	0.9387	0.9951	1.0000
LL	0.5676	0.8444	0.9429	0.9951	1.0000
SF	0.6256	0.8938	0.9676	0.9983	1.0000
S	0.5705	0.7716	0.8464	0.9116	0.9529
K	0.5343	0.8347	0.9407	0.9946	1.0000
χ^2	0.5261	0.7789	0.9087	0.9848	0.9999

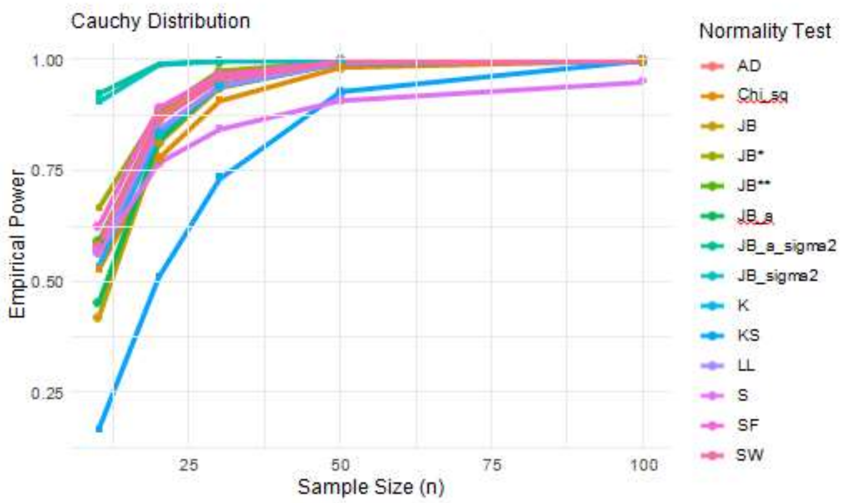


Figure 11: Empirical power of normality tests under Cauchy distribution.

Table 14: Empirical power of normality tests under the Uniform(0,1) distribution for various sample sizes

Test	n=10	n=20	n=30	n=50	n=100
JB**	0.0138	0.0014	0.0042	0.1873	0.9384
JB*	0.0420	0.0054	0.0037	0.0003	0.0000

JB_a	0.0000	0.0000	0.0020	1.0000	1.0000
$JB_{\sigma}2$	0.0000	0.0000	0.0000	0.0000	1.0000
$JB_{a,\sigma}2$	0.0000	0.0000	0.0000	0.0000	1.0000
SW	0.0829	0.1981	0.3810	0.7520	0.9970
AD	0.0524	0.1451	0.2776	0.5650	0.9469
KS	0.00000	0.00005	0.00045	0.00120	0.00890
JB	0.0027	0.0003	0.0001	0.0002	0.5621
LL	0.0642	0.0945	0.1388	0.2573	0.5895
SF	0.0513	0.0838	0.1725	0.4802	0.9658
S	0.0205	0.0060	0.0031	0.0020	0.0010
K	0.0740	0.2991	0.5690	0.8856	0.9988
χ^2	0.0897	0.0820	0.1088	0.1957	0.4631

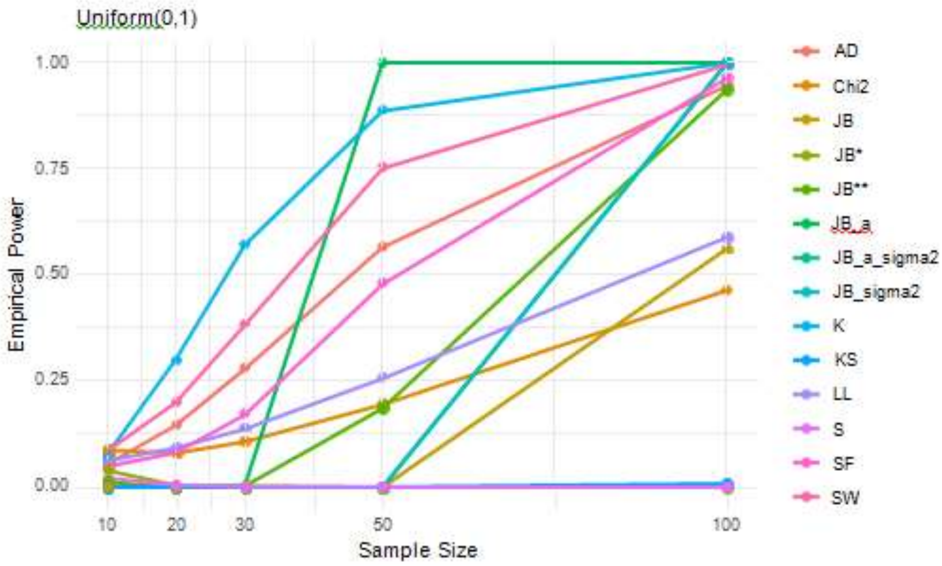


Figure 12: Empirical power of normality tests under Uniform(0,1) distribution.

Table 15: Empirical power of normality tests under Chi-Square(df=4) and Chi-Square(df=5) distributions for various sample sizes

	Chi-Square(df=4)					Chi-Square(df=5)				
Test	n=10	n=20	n=30	n=50	n=100	n=10	n=20	n=30	n=50	n=100
JB**	0.1931	0.3481	0.4920	0.7082	0.9717	0.1694	0.2984	0.4286	0.6288	0.9322
JB*	0.1906	0.2268	0.3706	0.4879	0.7273	0.1611	0.1848	0.3084	0.4011	0.6151
JB_a	0.0104	0.0708	0.1945	1.0000	1.0000	0.0036	0.0317	0.0943	1.0000	1.0000
$JB_{\sigma}2$	0.9325	0.9971	0.9998	1.0000	1.0000	0.9714	0.9994	0.9999	1.0000	1.0000
$JB_{a,\sigma}2$	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
SW	0.2390	0.5352	0.7544	0.9480	0.9998	0.1992	0.4443	0.6459	0.8856	0.9979
AD	0.1761	0.4345	0.6518	0.8841	0.9969	0.1432	0.3563	0.5368	0.7994	0.9875
KS	0.0007	0.0086	0.0228	0.0782	0.3333	0.0005	0.0061	0.0135	0.0466	0.2122
JB	0.0780	0.2892	0.4782	0.7632	0.9908	0.0624	0.2431	0.4044	0.6694	0.9727
LL	0.1699	0.3236	0.4650	0.6944	0.9565	0.1444	0.2661	0.3796	0.5845	0.8955
SF	0.2413	0.5033	0.7113	0.9259	0.9995	0.2029	0.4231	0.6095	0.8535	0.9958
S	0.2274	0.4724	0.6739	0.8961	0.9965	0.1952	0.4087	0.5870	0.8294	0.9893
K	0.1424	0.2372	0.3126	0.4221	0.6443	0.1224	0.2031	0.2666	0.3658	0.5598
χ^2	0.19(	0.2851	0.4102	0.6456	0.9496	0.1651	0.2142	0.3090	0.4922	0.8386

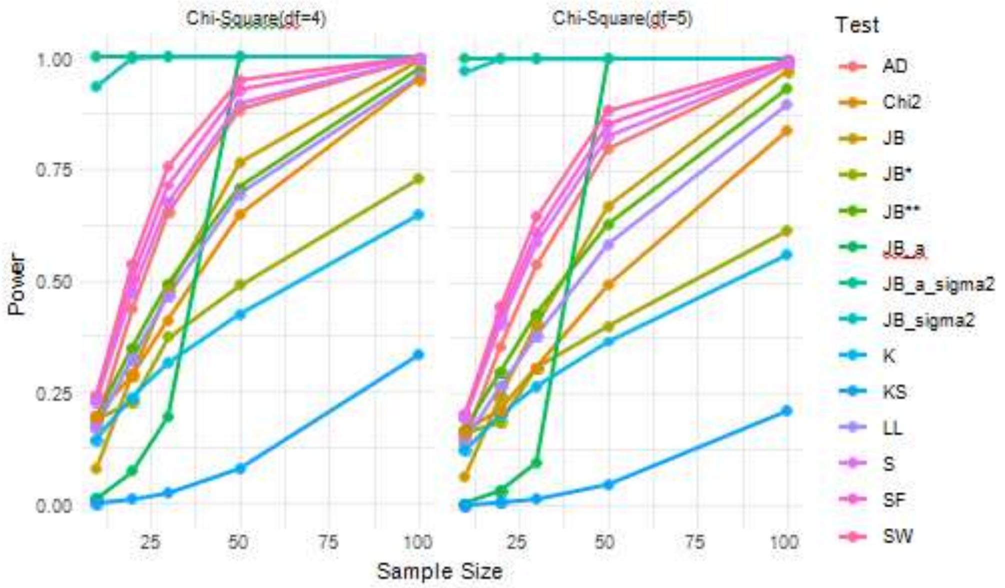


Figure 13: Empirical power of normality tests under Chi-Square(df=4) and Chi-Square(df=5) distributions.

Emperical study

Our initial dataset comprises for the purpose of determining the mechanical damage on corn seeds when subjected to compression up to the point of failure. Besides that, Mancera-Rico et al. (2016) considered the variable with the increase in corn seed length under the pressure stress. This quantity is call stress. These data shown in Table 16 represent 90 mm of the strain variable of corn seeds with floury endosperm and 8% of moisture. A histogram, qq-plot, and empirical cumulative distribution function (ecdf) of these observations is presented in the Figure 15a.

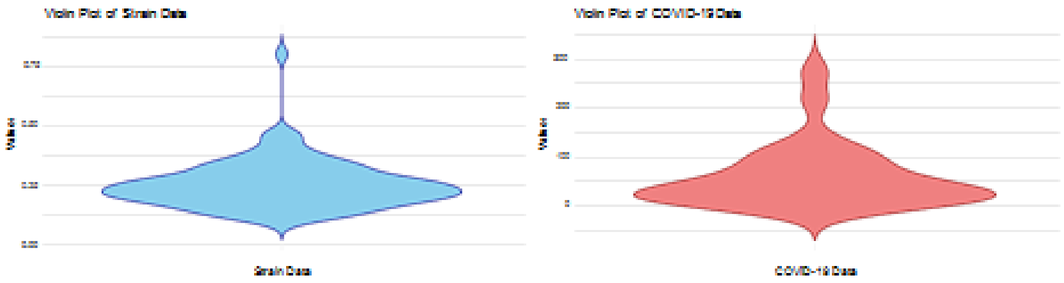
The Second set of data shown in Table 17 comprises of COVID-19 patients in Zimbabwe from July 1, 2020 to July 25, 2020. This dataset has been used in the study by the current author and others, as cited in Elbatal et al. (2024), and is freely accessible online at the World Health Organization's COVID-19 database. A histogram, qq-plot, and empirical cumulative distribution function (ecdf) of COVID-19 patients is presented in the Figure 16c. Both data violin plots are shown in Figure 14.

Table 16: The strain data.

0.293	0.274	0.80	0.262	0.270	0.284	0.177	0.463	0.257	0.212
0.192	0.336	0.22	0.371	0.208	0.287	0.343	0.261	0.246	0.276
0.262	0.269	0.226	0.331	0.267	0.231	0.329	0.246	0.465	0.168
0.164	0.170	0.193	0.270	0.242	0.369	0.242	0.206	0.227	0.226
0.307	0.325	0.166	0.118	0.145	0.225	0.210	0.130	0.103	0.232
0.257	0.099	0.249	0.116	0.183	0.355	0.147	0.128	0.193	0.237
0.128	0.186	0.448	0.160	0.282	0.197	0.400	0.213	0.196	0.272
0.386	0.213	0.165	0.215	0.192	0.147	0.126	0.186	0.322	0.201
0.415	0.223	0.287	0.331	0.234	0.130	0.318	0.322	0.185	0.357

Table 17: COVID-19 data.

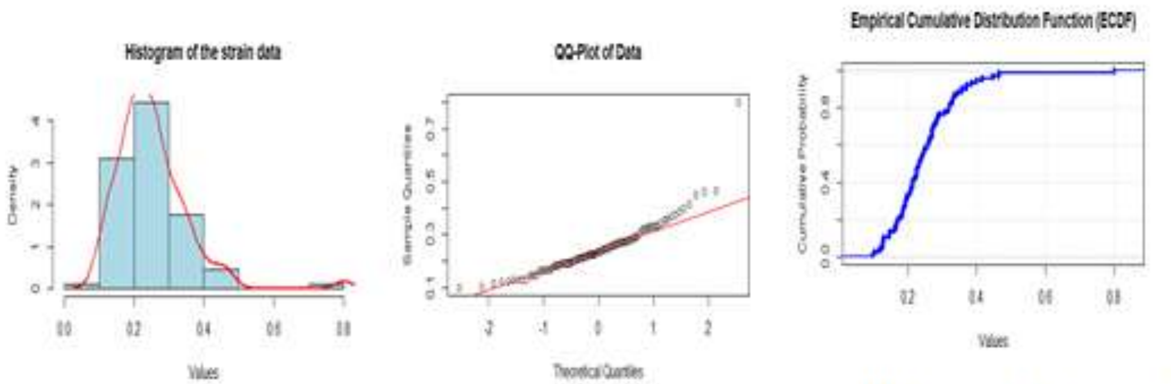
7	17	14	12	8	73	18	18	53	98
41	16	40	3	49	30	25	273	58	5
8	133	102	107	214	90				



(a) The strain data. (b) The COVID-19 data.

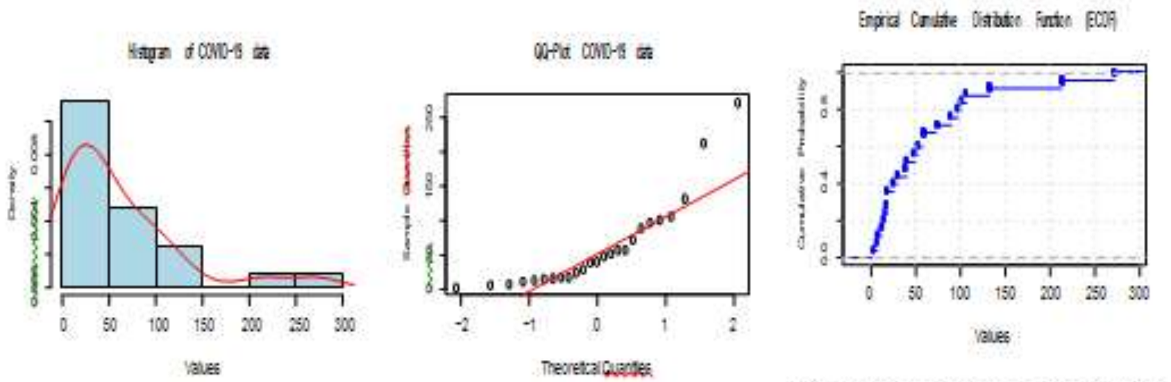
Figure 14: Visualizations of the violin plot: (a) strain data, (b) COVID-19 data

The violin plots of the strain data and the COVID-19 data visuallization shown in Figure 14 provide the shape of the distribution and identifies the preliminary measures of center and spread for both datasets. The strain data plot presents a density plot of the pressure values and a horizontal line at the median that gives an impression of how strain values are distributed and with areas that are broad at some values and thin at other revealing the likelihood of strains at such values. By the same token, the COVID-19 data plot should be relatively more erratic or positively skewed, with sharp peaks, humps or dips likely to indicate periods of high counts or otherwise. Comparing the two, the spread of the COVID-19 data might be broader and skewed or in other case have a wider symmetry as compared to the strain data which may be more symmetrical or consistent. The central lines or median are useful in showing a general location in which most values lie for the purpose of identifying departures of the central tendencies between the data sets.



(a) histogram. (b) qq-plot. (c) empirical cumulative distribution function (ecdf).

Figure 15: Visualizations of the strain data: (a) histogram, (b) qq-plot, and (c) ecdf.



(a) histogram. (b) qq-plot. (c) empirical cumulative distribution function (ecdf).

Figure 16: Visualizations of the COVID-19 data: (a) histogram, (b) qq-plot, and (c) ecdf.

Table 18: Normality test results

Data	SW	AD	KS	JB	LL	SF	S
covid data	5.218468e-05	2.307692e-05	2.344821e-01	4.116037e-06	7.316236e-03	1.171844e-04	3.458485e-04
strain data	1.554462e-07	6.666667e-06	2.029512e-01	0.000000e+00	6.642652e-03	5.191253e-07	6.299597e-09
Data	**K**	**$\chi 2$**	**JB****	**JB***	**JB_a**	**$JB_\sigma 2$**	**$JB_{a,\sigma}2$**
covid data	7.235438e-03	2.002912e-02	0.0017	0.0012	0.0003369875	0.0005862666	0.02943506
strain data	5.466807e-07	3.272148e-01	0.0000	0e+00	0.0000	0.0000	0.0000

The above empirical works reviewed the mechanical damage on corn seeds when compressed and data on COVID-19 patients in Zimbabwe. The coordinate data reflected the differences in the individual specimens under stress strain conditions shown in the histogram plot, qq-plot, and the ecdf displayed in Figure 16. These visualizations can be regarded as illustrating the distribution characteristics of the strain measurements, which was significant for determining the physical properties of corn seeds. Moreover, the COVID-19 data analysis carried out through statistical tests (refer to Figure 4) gave insights into the trends and results of this pandemic within the given period only.

Table 18 has condensed the results from the normality tests into one place which includes COVID and strain data results. Our proposed tests JB* and JB** compute p-values of 0.0012 and 0.0017 in the COVID data analysis which establishes robust evidence against normality distribution. The strain data fails to meet normality expectations because all modified Jarque–Bera tests including JB*, JB**, JB_a, $JB_\sigma 2$, and $JB_{a,\sigma}2$ generate zero level p-values. In contrast, other tests such as Kolmogorov–Smirnov and the chi-square test show weaker evidence against normality in these datasets. The evaluation reveals that our proposed tests (JB* and JB**) exhibit high sensitivity to abnormalities in skewness and kurtosis thus offering strong non-normality detection capabilities when traditional tests deliver uncertain findings.

Conclusion

Medical research needs valid statistical inference to make precise clinical choices because researchers handle patient health measurements including blood pressure and cholesterol levels and biomarker frequencies. Many clinical trials and epidemiological investigations and biomedical research studies depend on normality assumptions to perform their parametric methods effectively. Medical data from real-life situations deviate from normal distributions because of skewed data patterns combined with heavy-tailed distributions while containing anomalous values therefore compromising the accuracy of hypothesis testing alongside confidence interval estimation and predictive modeling.

A comprehensive evaluation of multiple normality tests covered Shapiro-Wilk, Anderson-Darling, Kolmogorov-Smirnov, Jarque-Bera, Lilliefors, Shapiro-Francia, D'Agostino's skewness, Anscombe-Glynn kurtosis, Pearson Chi-Square, and modified Jarque-Bera (JB_a,$JB_\sigma 2$,$JB_{a,\sigma}2$) tests as well as First Adapted Jarque-Bera (JB*) and Second Adapted Jarque-Bera (JB**). The study used wide-ranging simulation tests involving different data sample sizes and distribution shapes to show that typical normality assessment methods struggle with achieving valid error rates in medical datasets that contain non-standaard distributions. The results demonstrate that clinicians should adopt more superior methods for determining normality in medical datasets.

The research demonstrates that robust estimators in conjunction with bootstrap procedures within JB* and JB** tests produce superior performance than regular methods by controlling extreme values which yields better sample distribution normality assessments. The identified results generate important implications for medical research because irregular deviations from normality distributions affect treatment efficacy assessments and diagnostic precision and epidemiological prediction rates. The use of robust normality testing procedures enhances medical study statistical analyses while improving both decision accuracy and clinical knowledge.

The advancement of normality tests requires future research to integrate their enhanced versions within statistical software packages for medical research so they can become more commonly used. Research should move forward to test these methodologies on large clinical databases and medical practice conditions so healthcare practitioners can validate their performance in genuine healthcare settings. The development of better normality assessment procedures will enhance medical research statistical inference quality that leads to more dependable evidence-based healthcare delivery.

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The data utilized in this study are available upon reasonable request from the corresponding author. Due to institutional and confidentiality restrictions, some portions of the dataset may not be publicly accessible. Any additional information required for replication of the study can be provided upon request.

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